Topological crystalline insulators (TCIs) are recently discovered topological phase with robust surface states residing on high-symmetry crystal surfaces. Different from conventional topological insulators (TIs), protection of surface states on TCIs comes from point-group symmetry instead of time-reversal symmetry in TIs. The distinct properties of TCIs make them promising candidates for the use in novel spintronics, low-dissipation quantum computation, tunable pressure sensor, mid-infrared detector, and thermoelectric conversion. However, similar to the situation in TIs, the surface states are always suppressed by bulk carriers, impeding the exploitation of topology-induced quantum phenomenon. One effective way to solve this problem is to grow low-dimensional TCIs which possess large surface-to-volume ratio, and thus profoundly increase the carrier contribution from topological surface states. Indeed, through persistent effort, researchers have obtained unique quantum transport phenomenon, originating from topological surface states, based on controllable growth of low-dimensional TCIs. This article gives a comprehensive review on the recent progress of controllable synthesis and topological surface transport of low-dimensional TCIs. The possible future direction about low-dimensional TCIs is also briefly discussed at the end of this paper.
1. Introduction

Thermal dissipation, due to the scattering of carriers during transport process, is a common problem in current silicon-based electronic devices. The emerging topological insulators (TIs), a new quantum phase whose surface is conductive but interior is insulating, opens up a hopeful route to solve this issue.\textsuperscript{[1–3]} Owing to the relativistic effect (spin–orbit coupling) and topological protection from time-reversal symmetry, spin-locked current on the surface/edge of TIs is immune to any nonmagnetic impurities, which endows them with great application potential in low-dissipation electronic devices and quantum information processing. The exciting discovery and novel properties of TIs motivate scientists to search for new topological phase classified by other invariants. Topological crystalline insulators (TCIs), protected by point-group symmetry, are such kind of topological phase.\textsuperscript{[4,5]} So far, the experimentally confirmed TCIs are SnTe and its related alloy Pb$_{1-x}$Sn$_x$Te(Se) that possess high-symmetry rock-salt crystal structure. Each high-symmetry surface of TCIs accommodates four Dirac states. Interestingly, through controlling the crystal symmetry, the topological nature of TCIs can be transformed from nontrivial to trivial phase by strain and electrical field. Meanwhile, surface states properties of TCIs are highly tunable by composition and temperature. All above features indicate TCIs are promising for exploiting tunable electronic and spintronic devices. However, similar to TIs, surface states transport of TCIs is usually overwhelmed by bulk carrier.\textsuperscript{[6–8]}

In order to resolve this problem, researchers dedicated to growing low-dimensional TCIs since huge surface-to-volume ratio of low-dimensional TCIs can significantly enhance contribution of carrier transport from surface states. Especially, as topological nature varies from one high-symmetry facet to another, controlling the crystal planes orientation of TCIs nanostructures is crucial for probing unique surface states. Furthermore, based on low-dimensional TCIs, researchers successfully observed quantum transport phenomenon from surface states. It is worth noting that, although it is a short time after the first theoretical prediction of TCIs by Fu et al. in 2011,\textsuperscript{[4,5]} researchers have achieved great success on growth and surface states transport of low-dimensional TCIs. This paper comprehensively reviews the recent progress in both synthesis and surface topological transport of low-dimensional TCIs. We first introduce the basic principle of TCIs. Then, we summarize and analyze the newest results about synthesis and quantum transport of low-dimensional TCIs. The possible future directions about low-dimensional TCIs are also proposed in the end.

2. Fundamentals of TCIs

2.1. From TIs to TCIs

One of the big breakthroughs in condensed matter physics is the discovery of quantum Hall effect (QHE) in the 1980s.\textsuperscript{[9]} QHE occurs in 2D electrons system when an intense and perpendicular magnetic field is applied to the electrons to circulate in quantized orbits. As a result, the edge of samples is characterized by dissipationless current flows while the interior becomes inert. QHE is considered to be the first TIs because Hall conductivity $\sigma_{xy}$ equals integral multiples ($n$) of quantum conductance $e^2/h$ and $n$ is the topological invariant.\textsuperscript{[10]} However, the requirements of low-temperature and strong magnetic field for generating QHE strictly limit its practical application in electronic devices.

In the year of 2006, Zhang et al.\textsuperscript{[1,2]} theoretically predicted quantum spin Hall effect (QSHE) in 2D HgTe quantum well, in which strong spin–orbital coupling replaces the role of external magnetic field to force the current to move in one direction without back scattering. Such new TIs belong to a novel topological classification of a $Z_2$ index. Time-reversal invariant property protects the surface spin-polarized electron flow from the scattering of nonmagnetic impurities. This prediction was subsequently verified by experiments in 2007.\textsuperscript{[11]} Channel conductivity $\sigma_{xx}$ was observed to be quantized to $2e^2/h$ in zero magnetic field, proving the existence of gapless edge states in CdTe/HgTe/CdTe quantum well. This encouraging discovery profoundly boosts the research of TIs.

Soon 3D TIs such as Bi$_{1-x}$Sb$_x$, Bi$_2$Se$_3$, Bi$_2$Te$_3$, and Sb$_2$Te$_3$ with 2D surface gapless states are validated by both theory\textsuperscript{[12]} and experiments.\textsuperscript{[12,13]} It is worth emphasizing that Bi$_2$Se$_3$, Bi$_2$Te$_3$, and Sb$_2$Te$_3$ are ideal building blocks for the study of topological surface states when we take account of the following aspects:\textsuperscript{[6,8,14]} (1) they have simple chemical stoichiometric ratio, (2) they are of layered crystal structure that each covalently bonded quintuple layer interacts with each other by weak van der Waals forces, enabling the synthesis of few-layer nanoplates by conventional vapor deposition or mechanically exfoliated methods, and (3) their surface is terminated by a single Dirac cone with a relatively large bulk bandgap ($\approx 0.2–0.3$ eV) which makes it accessible to surface states even at room temperature.

Inspired by the TIs, theorists are committing themselves to find new type of TIs by other symmetry. Fu et al.\textsuperscript{[4]} first proposed that insulators with mirror symmetry, namely TCIs, could also obtain robust surfaces states. They subsequently present definite materials of TCIs that are SnTe and its related alloy Pb$_{1-x}$Sn$_x$Te(Se).\textsuperscript{[5]} The theoretical predication was immediately confirmed by three groups who detected the linear Dirac dispersion on mirror-symmetry surfaces of TCIs by angle-resolved photoemission spectroscopy (ARPES).\textsuperscript{[15–17]} The discovery of TCIs considerably extends the family of TIs. In starkly contrast to TIs, TCIs show differences in the aspects of (1) they are of highly symmetry crystal structures, (2) gapless metallic states only reside on those mirror-symmetry surfaces such as (100), (111), and (110), (3) Dirac cones on TCIs surface can be opened up by breaking symmetry, manipulating temperature, and tailoring compositions. Basic information about...
TIs and TCIs is listed in Table 1 from which we can clearly see peculiar characteristics of TCIs. Take Pb$_{1-x}$Sn$_x$Te as a representative example, it crystallizes in the form of cubic crystal structures. And it undergoes a band inversion and obtains topological protection when Sn content ($x$) reaches 0.38 at 9 K.\cite{15} Temperature also drives the conversion of Pb$_{1-x}$Sn$_x$Te from topological nontrivial insulator to trivial insulator when it exceeds the critical temperature ($T_c$). Intriguingly, external physical disturbs such as strain and electric field can open up the surface gapless Dirac states by destroying the crystal symmetry. These peculiar characteristics make TCIs a promising materials system in the applications of tunable spintronic devices.

### 2.2. Surface Electronics of TCIs

TCIs harbor four Dirac cones on each high-symmetry surface. Figure 1a presents the electronic structure of (100) surface in bulk SnTe.\cite{15} Two surface bands with opposite mirror eigenvalues cross each other and form a Dirac point along $TX$. Four Dirac points can be found on the four equivalent $TX$ as shown in Figure 1b. The surface Dirac states show a Lifshitz transition that the Fermi surface first exhibits two disconnected hole pockets outside $X$.\cite{18} And then they close to each other with the decrease of Fermi energy and touch each other. A large electron pocket with a hole inside is formed finally (Figure 1c). Another important feature of TCIs lies in the tunability of topological surface states through composition and temperature.\cite{17,19,21} Figure 1d shows the ARPES of Pb$_{1-x}$Sn$_x$Se (001) surface at various temperatures in the vicinity of X. It obviously presents that surface state is gapped above 100 K at which bottom of conduction band connects to top of valence band with the formation of a Dirac node.\cite{15} Composition dependence of topological surface states is also proved by Chen et al.\cite{22} Madhavan et al.\cite{23,24} further pointed out that the nontopological regime also host surface states. However, the weight of Dirac surface states decreases when Sn content ($x$) approaches the trivial phase, imparting the mass to the massless Dirac electrons. Zero-mass Dirac fermions protected by crystal symmetry were found to coexist with massive Dirac fermions.

Table 1. Fundamental information of TIs and TCIs. Insets: a) quantum Hall effect in 2D electron system with dissipationless edge states. b) Back scattering from nonmagnetic impurities in TIs surface is prohibited. Reproduced with permission.\cite{60} Copyright 2011, Nature Publishing Group. c) 2D helical surface states of 3D TIs. d) Energy dispersion of spin nondegenerate surface states on 3D TIs. Reproduced with permission.\cite{61} Copyright 2013, The Physical Society of Japan. e) Dirac-cone surface states on two different surface planes of (001)\cite{16,28} and (111).\cite{18} Reproduced with permission,\cite{18} Copyright 2013, The American Physical Society.

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<th>Materials</th>
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<th>Quantum spin Hall effect</th>
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<td>2D TIs</td>
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<td>Bi$_{1-x}$Sb$_x$, Bi$_2$Te$_3$, Bi$_2$Se$_3$, Ag$_2$Te, Bi$<em>2$(Se$</em>{1-x}$Te$_x$)$<em>3$, (Bi$</em>{1-x}$Sb$_x$)$_2$Te$_3$</td>
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electrons due to the symmetry-breaking distortion on the surface. And the magnitude of the symmetry-breaking distortion nearly keeps unchanged in the both topological and nontopological regimes.

Since metallic surface states of TCIs are protected by crystal symmetries instead of time-reversal symmetries, low-dissipation logic devices and pressure sensors can be developed based on topological surface states by breaking the crystal symmetry. Fu et al. first theoretically demonstrated topological transistor devices of TCIs thin film grown along the (001) direction.\[25,26\] They proposed the surface states of an 11-layer SnTe thin film will be gaped when applying a vertical electrical field. Figure 1e shows the surfaces electronic structures of SnTe (001) surface without application of electric filed. After a 0.1 V bias is applied across the thin film, the surface band opens up (Figure 1f). They further revealed that the ferroelectric-type structural distortion opened some or all of Dirac points, whereas strain moves the Dirac points to the Brillouin zone. And the perpendicular magnetic field generates the discrete Landau levels while in-plane magnetic field causes asymmetry between Dirac points.\[27\]

3. Controllable Growth of Low-Dimensional TCIs

Achieving controllable growth is always a very important step as well as a big challenge for any “new” material. This is especially true for TCIs in which fragile surface states may become undetected under the perturbation of defects and bulk state.\[17\] To avoid the disturbance of defects, high quality TCIs prepared by thermodynamic equilibrium synthesis method, like modified Bridgeman\[28\] and self-selecting vapor growth methods,\[17\] are used in the initial experimental studies. However, as we mentioned before, TCIs are topological insulators in which the gapless surface states are protected by mirror symmetry of the crystal.\[16\] In another word, possessing objects with large area of surfaces (surface states) of specific crystalline planes (mirror symmetry) is prerequisite if one wants to implement better experimental observation of possible topology-related phenomena. Taking this into consideration, synthesizing low-dimensional TCIs is one of the best choices. After its theoretical prediction in 2011,\[4\] many ways, such as molecular beam epitaxy (MBE) and vapor deposition method, have been employed to synthesize low-dimensional TCIs. In the following part, we will give a review on this rising field. We note that there are still other ways,\[29–31\] such as solution-phase synthesis method, to grow low-dimensional TCIs. But the products either are too small for device fabrication or have too bad crystalline quality for probing surface state. Hence, they will not be discussed in this section.

3.1. MBE for Thin Film

The first TCI proved by experiment is SnTe.\[16\] However, due to intrinsic Sn vacancies (usually p-type doped state),

Figure 1. a) Band structure and b) Fermi surface of SnTe (001) surface. c) A set of Fermi surface at different energy with a Lifshit transition. Reproduced with permission.\[5\] Copyright 2012, Nature Publishing Group. d) Temperature-dependent ARPES spectra in the vicinity of X. Reproduced with permission.\[15\] Copyright 2012, Nature Publishing Group. e) Gapless edge states of an 11-layer SnTe thin film. f) The edge state opens up when applying a perpendicular electrical field. Reproduced with permission.\[26\] Copyright 2014, Nature Publishing Group.
observing surface state of SnTe has been a challenge.\textsuperscript{[16]} As a result, synthesizing high-quality SnTe with few defects is the first task. MBE, by precisely controlling the deposition rate and compositions, is a promising candidate. In 2013, Yan et al. grow high-quality SnTe thin film on Si(111) substrates by MBE for the first time.\textsuperscript{[32]} Relying on the good controllability of MBE, thickness- and Pb doping-dependent properties of SnTe thin film are studied. Soon after, Assaf et al. synthesized SnTe film by MBE on BaF\textsubscript{2} (001) substrates.\textsuperscript{[33]} They found that high-quality film, then an increased Hall mobility and decreased carrier concentration, could be achieved through using higher growth temperature. To take a step further, Taskin et al. grow SnTe film using Bi\textsubscript{2}Te\textsubscript{3}, which has a lower lattice mismatch with SnTe, as substrates.\textsuperscript{[34]} Beyond this binary TCI, its ternary compound Pb\textsubscript{1−}xSn\textsubscript{x}Te is also an interesting material because its topological nature can be tailored by Sn content (x).\textsuperscript{[10]} According to this idea, Yan et al. synthesized Pb\textsubscript{1−}xSn\textsubscript{x}Te film with variable Sn content and found that there is a topological phase transition from trivial to nontrivial state while increasing Sn content.\textsuperscript{[22]}

3.2. Vapor Deposition Method

The thickness and composition of films grown by MBE are precisely controlled. However, it is expensive and time consuming. As compared with MBE, vapor deposition method is easier, cheaper and considered one of the most promising routes to productively synthesize nanostructured materials. For TCIs, the first trial is on SnTe. In 2013, through adjusting the temperature of substrates, Li et al. successfully synthesized SnTe nanomaterials with different morphologies by chemical vapor deposition (CVD).\textsuperscript{[35]} Combining with ab initio calculation, they found that it is the energy difference of crystalline planes that determines the morphologies (see Figure 2a). Specifically, when the substrates were placed in the relative higher temperature (here 645–675 °C), the Te rich condition, (100) planes have the lowest surface energy (left side of Figure 2a). As a consequence, SnTe microcubes (without Au catalyst, VS growth mode) and nanowires (with Au catalyst, VLS and VS growth mode) with (100) exposed planes were obtained. While on the occasion of lower temperature (525–625 °C), the Te in rich condition, (100) and (111) planes have a similar surface energy (right side of Figure 2a). As a result, complicated SnTe microcubes (without Au catalyst, VS growth mode) and zigzag nanowires (with Au catalyst, VLS and VS growth mode) which exposed both (100) and (111) planes were grown. Figure 2b,c shows the respective results. Almost at the same time, by controlling the experimental conditions in the CVD process, our group reported the synthesis of high-quality SnTe nanocrystals and nanowires with highly symmetry facets.\textsuperscript{[36,37]} Without the utilization of Au catalyst, the surface orientation and crystal shapes of SnTe micronanocrystals mainly depend on the growth temperature which decides the surface energy of SnTe micronanocrystals. However, when using Au nanoparticles as the catalyst, the 1D anisotropic growth is excited due to the induction of catalyst. Combining the effect of temperature, we obtained octahedron-attached SnTe nanowires and truncated octahedron-assisted SnTe nanowires. More recently, catalyst composition is found to have a strong impact on the morphologies of SnTe nanostructure.\textsuperscript{[36]} Zou et al. pointed out that liquid Au–Sn catalyst with high Sn content induced the thermally dynamic growth of SnTe triangular nanoplates with dominant [100] surfaces at high growth temperature region (500 °C). While SnTe nanowires with dominant [100] side surfaces were formed in the low-temperature region (450 °C) under the control of solid or quasisolod Au\textsubscript{5}Sn catalysts. In addition to the influence from surface energy of different crystal planes at given conditions, catalyst composition played the critical role in deciding the morphologies of SnTe nanostructures by affecting interface lattice match and chemical potential of Sn content. Compared with 3D bulks, 1D nanowire are favorable for probing surface state of TCIs due to their high surface-to-volume ratio.\textsuperscript{[39]} However, for a better performance, nanoplates with large top and bottom surfaces are more preferred. Having this idea in mind, Cha et al. grew SnTe nanoplates via the CVD method recently.\textsuperscript{[40]} Figure 2d–n gives their results. In detail, SnTe nanoplates with (100) or (111) planes as top and bottom surfaces were achieved when grown in a relative low temperature (around 300 °C). While in higher temperatures (350–450 °C), nanoribbons and nanowires would appear.

In the conventional vapor deposition method, substrates with dangling bonds, like SiO\textsubscript{2}/Si, are usually used. And vertical nanostructures with a variety of morphologies have been synthesized. But for 2D nanomaterials parallel to substrates, it turns out to be difficult because, as a result of the strong bonds, the adatoms become hard to diffuse on the top of substrates.\textsuperscript{[40]} Thanks to van der Waals epitaxy (vdWE) method, in which atomically smooth substrates (such as mica) are used as substrates, many nanomaterials with 2D morphologies have been synthesized successfully.\textsuperscript{[40–42]} For example, Li et al. prepared single-crystal topological insulators (Bi\textsubscript{2}Se\textsubscript{3} and Bi\textsubscript{2}Te\textsubscript{3}) nanoplates arrays using mica as substrates.\textsuperscript{[40]} However, previous works focused on growing 2D layered materials which have strong intrinsic driving force of 2D anisotropic growth due to their planar crystal structure. In the year of 2014, our group first proposed the growth of 2D nonlayered Te hexagonal nanoplates by vdWE on mica substrate.\textsuperscript{[43]} vdWE is advantaged in that (1) it allows large lattice mismatching between the target product and substrates, (2) excessive strain in the heterointerface is relaxed due to weak van der Waals interaction in the interface, and (3) van der Waals substrates facilitates the migration of adatoms and thus promotes the lateral growth of nanoplates. By performing the synthesis of SnTe and Pb\textsubscript{1−}xSn\textsubscript{x}Se nanostructures on mica substrates, we further put forward the general strategy for van der Waals epitaxial growth of 2D nonlayered semiconductor.\textsuperscript{[44]} Two conditions are required for the growth of 2D nonlayered materials by vdWE: (1) the nonlayered materials should have the driving force for the 2D anisotropic growth, which can be excited by optimizing the experimental conditions in CVD process, and (2) van der Waals materials such as mica and BN need to be used as the growth substrates. Figure 3 shows representative examples of van der Waals epitaxial
2D nonlayered material. Ultrathin 2D Pb$_{1-x}$Sn$_x$Se nanoplates with thickness ranging from 15 to 50 nm have been successfully synthesized on the surface of mica. Two growth conditions, in the CVD process, are thought to be the critical factors that affect the anisotropic growth of 2D TCIs. One is the substrates temperature that mainly determines the chemical activity of different crystal planes. In the case of our previous work about vdWE of 2D Pb$_{1-x}$Sn$_x$Se nanoplates on mica, {110} surfaces of Pb$_{1-x}$Sn$_x$Se showed higher activation at growth temperature of 550 °C compared with other facets, which leads to 2D anisotropic growth of Pb$_{1-x}$Sn$_x$Se nanoplates. The other important growth parameter is substrate surface chemistry property that influences the nucleation, migration of adatoms, interface stability, and thus the morphology of final products. Our previous work showed that, when we replaced layered mica with Si substrate, Pb$_{1-x}$Sn$_x$Se preferred to form microplates rather than nanoplates under the same experimental parameters as that on mica. This would be understood by the fact that (1) dangling bonds on surface of 3D bonded Si cause strong interaction between substrate and adatoms and thus increase the migration energy barriers of adatoms, (2) large lattice mismatch between Si (100) surface and Pb$_{1-x}$Sn$_x$Se ($\approx$10.4%) makes it unstable assuming Pb$_{1-x}$Sn$_x$Se epitaxially grows along surface of Si with planar geometry. Even anisotropic growth of layered materials such as MoS$_2$ is strongly affected by surface electronic properties of growth substrate. A recent work showed MoS$_2$ tended to form 1D nanobelt structure on Si instead of 2D nanoplates on SiO$_2$. The higher surface energy of Si (100 meV Å$^{-1}$) compared with SiO$_2$ (2.5 meV Å$^{-1}$) explains the occurrence of this growth behavior.

Figure 2. a) The free energies of the (100), (110), (111): Sn and (111): Te surfaces as functions of the relative Te chemical potential $\Delta\mu_{Te}$. Wulff constructions of the thermodynamic equilibrium SnTe crystals under the Te-lean and Te-rich conditions are shown in the inset. Wulff constructions b) and SEM images c) of SnTe nanostructures when Te is not in rich. Wulff constructions d) and SEM images e) of SnTe nanostructures when Te is in rich. The scale bars in bottom of (e) are 200 nm. Reproduced with permission. Copyright 2013, American Chemical Society. f) CVD growth schematic of SnTe nanoplates with SiO$_2$/Si used as substrates. g) SnTe unit cell. h) (100) cubic crystals, grown without Au catalyst. i,j) (100) nanoplates. k) (100) nanoribbon. l–n) Nanowires with <100> growth direction. o) (111) nanoplate and p) (111) nanoribbon. Reproduced with permission. Copyright 2014, American Chemical Society.
4. Surface Electronic Transport of Low-Dimensional TCIs

As mentioned before, because their huge surface-to-volume ratio notably magnifies the transport of carriers from surface states, low-dimensional TCIs are better candidates for detecting topological surface states by electrical transport. More intriguingly, it is easier to observe quantum interference phenomenon in TCIs nanostructures, which could be understood by the fact that (1) topological surface states have considerable mobility and (2) physical size of low-dimensional TCIs is comparable to phase coherent length of surface states. This section surveys the recent research results about topology-related electronic transport on low-dimensional TCIs.

4.1. Quantum Coherence Transport of Topological Surface States

It is reasonable to deduce that, compared with bulk counterpart, the wave nature of topological surface states in low-dimensional TCIs is more remarkable. In 2013, our group first discovered Aharonov–Bohm (AB) interference of surface Dirac electrons in SnTe nanowires. AB interference is the result of quantum interference of two partial electron waves along the edge of nanowires which encircle a certain magnetic flux. It was first used to characterize topological surface states of TIs nanostructures by Peng et al. in 2010. In our work, we additionally detected Altshuler–Aronov–Spivak (AAS) effect which arises from quantum interference of two interfering electron waves enclosing the magnetic flux once. Both AB and AAS effects deteriorate with the increase of temperature because of the disturbing from thermal excitation. We further found SnTe nanowire exhibits pronounced Shubnikov–de Haas (SdH) oscillations under the exposure of vertical magnetic field. This work is the first evidence of topological surface states in TCIs nanostructures by magneto-transport. Almost simultaneously, topological surface states transport was confirmed in SnTe thin film by Ando et al. In this work p-type SrSiTe thin film was grown on n-type Bi2Te3 thin film.
by MBE. They observed SdH oscillation of Dirac fermions residing on the SnTe (111) surfaces due to a downward band bending on the free SnTe surface (Figure 4c). Based on SnTe nanoplates, Cha et al. reported a structural phase transition of SnTe that it transforms from rock salt at high temperature to rhombohedral structure at low temperature. However, the work has not presented the weak antilocalization (WAL) effect of topological surface states, which would be because the SnTe nanoplates are too thick to enhance transport of surface states.

Multiple Dirac nodes on each high-symmetry surface bring more complexity to surface transport of TCIs nanostructures. In addition to the coupling between bulk and surface states, the hybrid of different Dirac states strongly affects the numbers of the transport channels. Heiman et al. carefully investigate the valley coupling of degenerate TCIs surface in SnTe thin film. As shown in Figure 4d, the numbers of carrier valleys (2α) extracted from the HLN model change with Fermi level (E_F). At low E_F, no bulk sates are involved in the charge transport and the Fermi surface contains four
Dirac surface valleys. $2\alpha = 8$ is expected to obtain in this case. As the Fermi level increases to the point of Lifshitz transition ($E_F = 0.05$ eV), pairs of Dirac surface cones merge into four pairs of concentric energy contours. Below this point, the inner Dirac cone and the outer Dirac cone host opposite chirality, which makes the quantum coherent transport in this case extremely complicated. In addition, the contribution of bulk states to transport becomes more and more important, the trivial bulk states swallow the inner Dirac cone, thus reducing the $2\alpha$ to smaller than 2. As the $E_F$ decrease to 0.2–0.5 eV, surface and bulk states coexist and a maximum of $2\alpha = 4$ can be obtained. With Fermi level exceeding 0.5 eV, the $2\alpha$ sharply decreases which would be due to the fact that the shape of the surface valleys changes again in the deep Fermi level. Figure 4e further shows that the numbers of transport channel linearly decrease with the increase of phase coherence length ($L$). The smaller experimental $2\alpha$ means the stronger valley coupling and thus results in a larger coherence length. The valley coupling may result from the scattering between top and bottom surfaces of SnTe thin film. And, given the valley degeneracy of SnTe, the coupling also likely comes from the scattering between two Dirac states on the same surface. The coupling between bulk and surface states has also been observed by Kuroda et al. who reported that both the numbers of transport channel and phase coherence length decrease with increase of temperature. This is because that thermal excited carriers take part in the transport and interact with the surface states at higher temperature. The above two works are crucial for understating the mechanism of surface states transport.

4.2. Surface Transport Modulation

Unique properties of TCIs provide more degree to control the surface states of low-dimensional TCIs. First, the Sn content ($x$) of Pb$_{1-x}$Sn$_x$Te(Se) has a strong impact on the surface electronic structure. For Pb$_{1-x}$Sn$_x$Te, when Sn content goes beyond 0.38 at 9 K, it undergoes a topological phase transition from trivial to nontrivial. Our group first demonstrated this characteristic in Pb$_{1-x}$Sn$_x$Te nanowires by electric transport.[52] By conducting the magneto-transport of Pb$_{1-x}$Sn$_x$Te nanowires, we observed the weak localization (WL) effect in PbTe nanowires while WAL effects in Pb$_{0.5}$Sn$_{0.5}$Te and Pb$_{0.2}$Sn$_{0.8}$Te nanowires. The PbTe nanowire exhibits the semiconductive behavior of electrical transport. The thermal activation energy ($E_a$), extracted from $R = e^{E_a/k_B T}$ at high temperature (260–133 K), is about 8.2 meV, where $k_B$ is the Boltzmann constant. The change of magneto-conductance at 2 K displays a sharp downward cusp near zero magnetic field, which is a signature of WL effect (Figure 5a). However, the Pb$_{0.2}$Sn$_{0.8}$Te shows metallic transport behavior and WAL effect that the curve of $AG$ versus magnetic field $B$ shows upward cusp around zero magnetic field (Figure 5b). The angle-traced magneto-transport confirms that the WAL effects originate from the 2D surface states. The topological phase transition by composition engineering has also been realized by Xiu et al. in Pb$_{1-x}$Sn$_x$Se thin films. They also found that strong electron–electron interaction caused the large resistance of Pb$_{0.75}$Sn$_{0.25}$Se thin films. Except the composition, surface states of TCIs can be modulated by thickness and gate voltage. As the thickness of TCIs thin film reaches the critical value at which top surface and bottom surface couple with each other, the resistance of thin film will dramatically increase due to the absence of topological surface states in this case. Xiu et al. further pointed out the critical value for Pb$_{1-x}$Sn$_x$Te thin film is 10 nm. The resistance of 10 nm Pb$_{1-x}$Sn$_x$Se thin film is about one order of magnitude larger than that of 16 nm Pb$_{1-x}$Sn$_x$Se thin film. They further performed the gate voltage modulation in the surface states of 16 nm Pb$_{0.93}$Sn$_{0.07}$Se thin films at 2 K. As shown in Figure 5c, the p-type Pb$_{0.9}$Sn$_{0.1}$Se thin films shows WAL effects by applying negative bias. The Berry phase $\phi$ is related to the Fermi level by

$$\phi = \pi \left[ 1 - \left( \Delta (2E_F) \right) \right]$$

where $\Delta$ is the gap size. The Fermi level moves toward valence band as the gate voltage decreases from positive value to negative value. And thus $E_F$ becomes larger and Berry phase approaches $\pi$. Interestingly, the competition between quantum interference ($\sigma^G$) and the electron–electron interaction ($\sigma^e$) also leads to the transition from WAL to WL effect. As shown in Figure 5d, at low temperature, the $\sigma^e$ serves as the dominant role and thus leads to the WL behavior due to the trivial Berry phase of surface states. However, $\sigma^e$ becomes the main factor that affects the conduction correction at high temperature, giving rise to the WAL effect.

If introducing the exotic elements, TCIs nanostructures will exhibit new electronic properties. In-doped SnTe has been confirmed to topological superconductor which hosts Majorana fermions.[54] Majorana fermions are charge-neutral particles whose antiparticles are themselves. They are attractive because of their great potential for using as a qubit of fault-tolerant topological quantum computing.[55] Superconductor Cu$_2$Bi$_2$Se$_3$ was found to present unconventional superconductivity in its point-contact spectra. Sn$_{1-x}$In$_x$Te also holds signatures of unconventional superconductivity which are important for finding massless Majorana fermions.[56] Superconductivity of Sn$_{1-x}$In$_x$Te has recently been confirmed by the Cha group[57] and the Ando group[58] individually by synthesizing Sn$_{1-x}$In$_x$Te nanoplates and fabricating their Hall devices. Cha et al. further pointed out In doping will profoundly enhance the surface states of Sn$_{1-x}$In$_x$Te nanoplates since In doping decreases the bulk mobility.[59]

At the end of this section, we want to briefly discuss the thermoelectric conversion properties of TCIs. Although it is still unclear whether topological surface properties of TCIs have relation with their thermoelectric nature, we noticed that almost all TCIs as well as TIs are excellent thermoelectric materials.[54] In this regard, Zhang et al. first demonstrated diameter dependence of thermoelectric properties of individual SnTe nanowires.[59] They found the thermopower enhances a magnitude of two orders with the decrease of diameter from ~913 to ~218 nm, while, due to the increasing boundary scattering and phonon-defect scattering, the thermal conductivity is notably reduced with the decrease of...
nanowire diameter. As a result, the simultaneous increase of thermopower and suppression of thermal conductivity results in an improved figure of merit $ZT$.

5. Prospects

Despite great progress in synthesis and surface transport of TCIs nanostructures, the depth and width of research on low-dimensional TCIs are far less than that on conventional TIs nanostructures. Previous research on conventional TIs nanostructures supplies a paradigm for us to exploit novel quantum behavior and potential applications of TCIs nanostructures. Like TIs, high-speed and low-dissipation electronic devices are thought to be one of the most promising applications for TCIs nanostructures.[60] Considering the ultrahigh surface mobility of TCIs,[34,36] ultrafast logic devices with low-thermal dissipation would be developed based on top gate and back gate modulation.

What is more, since crystal symmetry warrants the surface states of TCIs, new type of logic device can be realized by applying vertical electrical field in a dual-gated field-effect transistor. The strong vertical electrical field breaks the crystal symmetry and thus opens up the surface Dirac cones. Furthermore, since crystal symmetry of TCIs can also be destroyed by strain, highly sensitive pressure sensors are expected to achieve in TCIs nanostructures. However, the above two applications rely on ultrathin TCIs nanoplates with high quality, which is an important opportunity as well as a big challenge for materials scientist.

In order to realize surface states-based electronic devices, one must lower the bulk carrier density. One way is to synthesize the low-dimensional TCIs.[61] Among various nanostructures, due to the dominant top and bottom surfaces, 2D nanoarchitectures with distinct mirror-symmetry facets are the best construction for exploring surface states. However, TCIs are different from conventional TIs with layered structures and time-reversal symmetry. TCIs are cubic crystal structure and the surface states only reside on high symmetry surfaces. Few-layer conventional TIs such as Bi$_2$Se$_3$ and Bi$_2$Te$_3$ have been successfully grown by CVD due to its strong intrinsic driving force for 2D anisotropic growth. However, it is difficult to do this for TCIs as a result of its nonlayered crystal structures. Meanwhile, one needs to tailor the surface of TCIs nanostructures to high symmetry crystal planes. Although our group have grown Pb$_{1-x}$Sn$_x$Se nanoplates with distinct (100) surfaces by vdWE, ultrathin SnTe and Pb$_{1-x}$Sn$_x$Te nanoplates are not yet reported. Meanwhile, formation of impurities in the TCIs nanostructures is inevitable during the growth and device fabrication process. For example, Pb$_{1-x}$Sn$_x$Se thin film and nanoplates are usually p-type doping caused by cation vacancies. Cations compensation may minimize the density of cation vacancies. One can further tune the mole ratio of Pb to Sn in ternary TCIs such as Pb$_{1-x}$Sn$_x$Se(Te). Similar to the work on (Bi$_{1-x}$Sb$_x$)$_2$Te$_3$ nanoplates, this way can push the Fermi level to the middle of bandgap and thus profoundly decrease the bulk states. Hybrid structure based on TCIs is also attractive for chemist and material scientist.[62,63] Interfacing TCIs nanostructures with superconductor, ferromagnetic insulator, and insulator will bring exotic properties.[64]

Figure 5. a) Dominated WL effect of PbTe nanowire at 2 K, inset is the SEM image of four-terminal device with scale bar of 2 µm. b) Temperature-dependent magnetoconductance of Pb$_{0.5}$Sn$_{0.5}$Te nanowire. Reproduced with permission.[52] Copyright 2015, American Chemical Society. c) Gate voltage-modulated surface transport of 16 nm Pb$_{0.9}$Sn$_{0.1}$Se thin film at 2 K. d) Change of magnetoconductance of 16 nm Pb$_{0.9}$Sn$_{0.1}$Se thin film at various temperatures. Reproduced with permission.[53] Copyright 2015, American Chemical Society.
For example, interface between superconductor and TCIs is predicted to generate Majorana fermions.

6. Conclusion

TCIs have been the star materials in condensed matter physics. The marvelous topological surface states, guaranteed by crystal symmetry, render scientists an opportunity for developing fundamental physics as well as low-dissipation electronic devices. Compared with the bulk counterpart, low-dimensional TCIs, due to their large surface to volume ratio, are more ideal system for exploiting surface states. This paper comprehensively summarizes the recent progress in controllable growth of low-dimensional TCIs. The device applications based on TCIs nanostructures are also carefully reviewed. Although this paper covers only the very tip of the iceberg, the intriguing properties of TCIs will excite the interest of a broad research community. It is worth noting, compared with TIs, much work on TCIs nanostructures has been left to do. For example, it has not yet elucidated how strain controls surface states transport. The unique electronic properties of TCIs nanostructures such as multiple surface states have not been very well documented either. We believe low-dimensional TCIs will bring us more exciting breakthroughs in the near future.

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